Exercise 32

Show that the function is continuous on its domain. State the domain.

$$g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}$$

Solution

The numerator, $\sqrt{x^2 - 9}$, is a square root function, so it's continuous at all numbers in its domain. The denominator, $x^2 - 2$, is a polynomial function, so it's continuous at all numbers in its domain: $(-\infty, \infty)$. The ratio of these functions given by $g(x)$ is continuous at the numbers that both the numerator and the denominator are continuous at, provided that the denominator is not zero. Find the domain of the numerator.

$$x^2 - 9 \geq 0$$

$$x^2 \geq 9$$

$$x \leq -3 \quad \text{or} \quad x \geq 3$$

Then find where the denominator is zero.

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$\sqrt{x^2} = \sqrt{2}$$

$$|x| = \sqrt{2}$$

$$x = \pm\sqrt{2} \approx \pm 1.414$$

Neither $-\sqrt{2}$ nor $\sqrt{2}$ are less than -3 or greater than 3, so the numerator's domain doesn't need to be modified. The domain of $g(x)$ is therefore

$$x \leq -3 \quad \text{or} \quad x \geq 3.$$

This is reflected in the graph of $g(x)$ versus x.